Spontaneous Symmetry Breaking, Non-minimal Coupling, and Cosmological Constant Problem

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Abstract

Treating the gravitational field as a dynamical field, we study the spontaneous symmetry breaking induced by a scalar field under its self-interaction and non-minimal interaction with gravity in four dimensional space-time. In particular, we explore the feasibility of inducing spontaneous symmetry breaking after introducing the non-minimal coupling, and discuss briefly the cosmological constant problem corresponding to the phase transition associated with the spontaneous symmetry breaking.

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1 Introduction

In physics, the standard way of implementing spontaneous symmetry breaking (SSB) is by introducing a scalar field with a potential like

$$V(\Phi) = -\frac{1}{2}\mu^2\Phi^2 + \frac{1}{4}\lambda\Phi^4, \tag{1}$$

which entails multiple true vacua (at $|\Phi| = \sqrt{\mu^2/\lambda}$).[1] In the process of choosing one among these true vacua, SSB may be induced. We note that the negative $(mass)^2$ Φ^2 term and $\lambda\Phi^4$ term in Eq. (1) together play a crucial role in making the potential entail multiple true vacua. As the scalar field rolls down to one of the true vacua from the origin, some phase transition may happen, and accordingly, its "latent heat" (with the amount of $|\delta V| = \mu^2/(4\lambda)$) will be released to the "thermal energy". In the framework of quantum field theory, the "latent heat" is the difference of vacuum energy in different phases, and the "thermal energy" is the energy of particles produced along with the phase transition. As pointed out by Zeldovich [2] (see also [3]), the vacuum energy will contribute to the effective cosmological constant. Thus, if we need a comparatively small effective cosmological constant (for the indication from various astronomical observations, for instance, the supernova distance measurements [4, 5]) after the phase transition, we must fine tune the parameters: μ and λ, and the original cosmological constant Λ_0 (before the phase transition), such that

$$\frac{1}{\kappa}\Lambda_0 \simeq \frac{\mu^4}{4\lambda}, \tag{2}$$

where $\kappa \equiv 8\pi G$, and G is the gravitational constant. Such a fine tuning is one of various versions of the cosmological constant problem.

The *non-minimal coupling* between scalar fields and gravity has been introduced in various topics in cosmology (see [6, 7] and references therein) and quantum field theories (for a review, see [8] and references therein). In the framework of quantum field theory in curved space-time, the non-minimal coupling can be introduced by the requirement of renormalizability [9] (for a review, see [10]). In particular, the role of the non-minimal coupling in SSB and phase transitions in four dimensional space-time has been widely discussed (for a review, see [8]): That the non-minimal coupling with the 'external' gravitational field may lead to SSB has been pointed out [11]. It has also been noted that SSB and phase transitions can be induced by curvature via the non-minimal coupling with the 'external' gravitational field [12], and moreover, the curvature can be a symmetry-breaking factor [13]. We note that, in the work mentioned above, the effect of the curved space-time is taken into account by introducing an 'external' gravitational field, that is, the metric tensor of the space-time is treated as a background. This is different from the usage of the gravitational field in our recent work to be discussed below.

Recently we study the influence of the non-minimal coupling with the *dynamical* gravitational field on SSB in the space-time of an arbitrary dimension [14], where we keep only the negative $(mass)^2$ Φ^2 term, but discard $\lambda\Phi^4$ term in the potential (Eq. (1))used in the standard SSB scenario. In this paper, we take *both* two terms in Eq. (1) into consideration,

[1] A famous example is the Higgs mechanism in the Standard Model to break the SU(2)×U(1) gauge symmetry spontaneously [1].

and focus on *four* dimensional space-time. In particular, we will consider a universe which is dominated by a scalar field and possibly by a cosmological constant, and show the feasibility of inducing SSB in this setup. We will also discuss briefly the cosmological constant problem corresponding to the phase transition associated with SSB.

2 The Basics

We consider a four dimensional universe, which is dominated by a scalar field, a cosmological constant, and gravity, and described by the following action:

$$S = -\frac{1}{2\kappa}\int d^4x\sqrt{g}\left(\mathcal{R}+2\Lambda\right) + \int d^4x\sqrt{g}\mathcal{L}_\Phi, \tag{3}$$

where $\kappa \equiv 8\pi G$ (G: gravitational constant), g is the absolute value of the determinant of the metric tensor $g_{\alpha\beta}$, Λ is the cosmological constant, and $\mathcal{L}_\Phi$ is the Lagrangian density of the scalar field Φ. We would like to introduce the non-minimal coupling into the scalar-field potential (or Lagrangian density $\mathcal{L}_\Phi$), including both two terms in Eq. (1) introduced in the standard SSB scenario. Accordingly, we consider

$$\mathcal{L}_\Phi = \frac{1}{2}g^{\alpha\beta}\left(\partial_\alpha\Phi\right)\left(\partial_\beta\Phi\right) - V_\mathcal{R}\left(\Phi\right), \quad \alpha,\beta = 0,1,2,3 \tag{4}$$

$$V_\mathcal{R}\left(\Phi\right) = \frac{1}{2}\xi\mathcal{R}\Phi^2 - \frac{1}{2}\mu^2\Phi^2 + \frac{1}{4}\lambda\Phi^4, \tag{5}$$

where the term $\frac{1}{2}\xi\mathcal{R}\Phi^2$ in the scalar-field potential $V_\mathcal{R}(\Phi)$ is the non-minimal coupling term, which couple the scalar field Φ with gravity via the Ricci scalar $\mathcal{R}$ with a coupling constant ξ to be positive in our consideration.[2]

The variation of the action in Eq. (3) with respect to the scalar field Φ and the metric tensor $g^{\alpha\beta}$ yields the field equation of Φ:

$$\Phi^{;\gamma}{}_{;\gamma} + \left(\xi\mathcal{R} - \mu^2 + \lambda\Phi^2\right)\Phi = 0, \tag{6}$$

and the Einstein equations:

$$G_{\alpha\beta} - \Lambda g_{\alpha\beta} = \kappa\left\{\left(\partial_\alpha\Phi\right)\left(\partial_\beta\Phi\right) - \mathcal{L}_\Phi^{(0)}g_{\alpha\beta} - \xi\Phi^2 G_{\alpha\beta} + \xi\left(\Phi^2\right)_{;\alpha;\beta} - \xi\left(\Phi^2\right)^{;\gamma}{}_{;\gamma}\,g_{\alpha\beta}\right\}, \tag{7}$$

where

$$\mathcal{L}_\Phi^{(0)} = \frac{1}{2}g^{\alpha'\beta'}\left(\partial_{\alpha'}\Phi\right)\left(\partial_{\beta'}\Phi\right) + \frac{1}{2}\mu^2\Phi^2 - \frac{1}{4}\lambda\Phi^4, \tag{8}$$

$G_{\alpha\beta}$ is the Einstein tensor, and the semicolon ';' denotes the 'covariant derivative'. (We note that the non-minimal coupling term is not included in $\mathcal{L}_\Phi^{(0)}$.) The term $-\xi\Phi^2 G_{\alpha\beta}$ in

[2]The convention for the metric tensor in this paper is:

$$g_{\alpha\beta} = (+,-,-,-)\,.$$

the brace in Eq. (7) will modify the Einstein equations, the gravitational constant G (also κ), and the cosmological constant Λ as follows:

$$G_{\alpha\beta} - \Lambda' g_{\alpha\beta} = \kappa_{eff} \left\{ (\partial_\alpha \Phi)(\partial_\beta \Phi) - \mathcal{L}_\Phi^{(0)} g_{\alpha\beta} + \xi \left(\Phi^2\right)_{;\alpha;\beta} - \xi \left(\Phi^2\right)^{;\gamma}_{\;;\gamma} g_{\alpha\beta} \right\}, \tag{9}$$

where the 'modified cosmological constant' Λ' and the 'effective gravitational constant' G_{eff}, as well as κ_{eff}, are defined by

$$\Lambda' \equiv \frac{\Lambda}{1+\kappa\xi\Phi^2} \tag{10}$$

$$8\pi G_{eff} \equiv \kappa_{eff} \equiv \frac{\kappa}{1+\kappa\xi\Phi^2}. \tag{11}$$

Taking trace of both sides of the *modified* Einstein equations (9) gives us the Ricci scalar $\mathcal{R}$ as a function of the scalar field Φ and its covariant derivatives:

$$\mathcal{R} = \frac{\kappa}{1+\kappa\xi\Phi^2} \left\{ -4 \left(\frac{1}{\kappa}\Lambda - \frac{1}{2}\mu^2\Phi^2 + \frac{1}{4}\lambda\Phi^4 \right) + \Phi^{;\gamma}\Phi_{;\gamma} + 3\xi \left(\Phi^2\right)^{;\gamma}_{\;;\gamma} \right\}. \tag{12}$$

As shown in the above equation, the Ricci scalar $\mathcal{R}$, by which the scalar field Φ is coupled with gravity, strongly depends on the scalar field. Such a strong dependence will make significant influence on the inducement of SSB to be discussed in the next section.

3 Multiple True Vacua

The existence of multiple true vacua is the key element of implementing SSB. For exploring the existence of multiple true vacua entailed by the action in Eq. (3) or the potential $V_\mathcal{R}(\Phi)$ in Eq. (5), we need to find out the constant solutions for the scalar field Φ and explore their stability (since a vacuum state is usually corresponding to a stable constant-field solution).

For constant Φ, the field equation of Φ (6) and *modified* Einstein equations (9) become

$$\left(\xi\mathcal{R} - \mu^2 + \lambda\Phi^2\right)\Phi = 0, \tag{13}$$

$$\begin{aligned} G_{\alpha\beta} &= \Lambda' g_{\alpha\beta} + \kappa_{eff} \left(-\frac{1}{2}\mu^2\Phi^2 + \frac{1}{4}\lambda\Phi^4 \right) g_{\alpha\beta} \\ &= \frac{\kappa}{1+\kappa\xi\Phi^2} \left(\frac{1}{\kappa}\Lambda - \frac{1}{2}\mu^2\Phi^2 + \frac{1}{4}\lambda\Phi^4 \right) g_{\alpha\beta} \\ &= \kappa_{eff} V_{eff} g_{\alpha\beta} = \Lambda_{eff} g_{\alpha\beta}, \end{aligned} \tag{14}$$

where the '*effective* potential energy (or vacuum energy)' V_{eff} and '*effective* cosmological constant' Λ_{eff} are defined by

$$V_{eff} = \frac{1}{\kappa}\Lambda - \frac{1}{2}\mu^2\Phi^2 + \frac{1}{4}\lambda\Phi^4, \tag{15}$$

$$\Lambda_{eff} = \frac{\kappa}{1+\kappa\xi\Phi^2} \left(\frac{1}{\kappa}\Lambda - \frac{1}{2}\mu^2\Phi^2 + \frac{1}{4}\lambda\Phi^4 \right) = \kappa_{eff} V_{eff}. \tag{16}$$

Taking trace of both sides of Eq. (14) gives us the Ricci scalar $\mathcal{R}$ as a function of the scalar field Φ:

$$\mathcal{R} = -\frac{4\kappa}{(1+\kappa\xi\Phi^2)}\left(\frac{1}{\kappa}\Lambda - \frac{1}{2}\mu^2\Phi^2 + \frac{1}{4}\lambda\Phi^4\right). \tag{17}$$

Using the above equation, Eq. (13) becomes

$$\left[-\frac{4\kappa\xi}{(1+\kappa\xi\Phi^2)}\left(\frac{1}{\kappa}\Lambda - \frac{1}{2}\mu^2\Phi^2 + \frac{1}{4}\lambda\Phi^4\right) - \mu^2 + \lambda\Phi^2\right]\Phi = 0. \tag{18}$$

Then we can obtain the constant solutions for the scalar field Φ:

$$\Phi = \Phi_{(c)} = 0,\ \pm v \tag{19}$$

where v is defined by

$$v^2 = \frac{4\xi\Lambda + \mu^2}{\lambda + \kappa\xi\mu^2} \quad > 0 \quad \text{for} \quad \xi\Lambda > -\frac{1}{4}\mu^2, \quad \text{if} \quad \lambda > -\kappa\xi\mu^2. \tag{20}$$

The corresponding metric tensor $g^{(c)}_{\alpha\beta}$ (the solution(s) of the modified Einstein equations (14) corresponding to $\Phi = \Phi_{(c)}$) will be the metric tensor of de Sitter or anti-de Sitter space-time, depending on the '*effective* cosmological constant' Λ_{eff} is positive or negative.

Now we need to explore the stability of these three constant-Φ solutions ($\Phi = \Phi_{(c)}$, $g_{\alpha\beta} = g^{(c)}_{\alpha\beta}$). Considering small variations around these solutions:

$$\begin{cases} \Phi = \Phi_{(c)} + \delta\Phi \\ g_{\alpha\beta} = g^{(c)}_{\alpha\beta} + \delta g_{\alpha\beta} \quad \longrightarrow \quad \mathcal{R} = \mathcal{R}_{(c)} + \delta\mathcal{R} \end{cases}, \tag{21}$$

where $\mathcal{R}_{(c)}$ is the Ricci scalar for $\Phi = \Phi_{(c)}$, the field equation of Φ (6) becomes (up to $\mathcal{O}(\delta\Phi, \delta\mathcal{R})$):

$$(\delta\Phi)^{;\gamma}{}_{;\gamma} + \left(\xi\mathcal{R}_{(c)} - \mu^2 + 3\lambda\Phi^2_{(c)}\right)\delta\Phi + \xi\Phi_{(c)}\delta\mathcal{R} = 0, \tag{22}$$

and the equation for the Ricci scalar $\mathcal{R}$ (12) becomes (up to $\mathcal{O}(\delta\Phi, \delta\mathcal{R})$):

$$\delta\mathcal{R} = 0 \quad \text{for} \quad \Phi_{(c)} = 0\,, \tag{23}$$

$$\delta\mathcal{R} = \frac{2\kappa}{1+\kappa\xi\Phi^2_{(c)}}\left(\mu^2 - \lambda\Phi_c^2\right)\Phi_{(c)}\delta\Phi + 6\xi\Phi_{(c)}\,(\delta\Phi)^{;\gamma}{}_{;\gamma} \quad \text{for} \quad \Phi_{(c)} = \pm v. \tag{24}$$

Using above equations and Eq. (17), we can obtain the field equations of $\delta\Phi$ from Eq. (22): For $\Phi_{(c)} = 0$,

$$(\delta\Phi)^{;\gamma}{}_{;\gamma} + \left[-\left(4\xi\Lambda + \mu^2\right)\right]\delta\Phi = 0, \tag{25}$$

where

$$-\left(4\xi\Lambda + \mu^2\right) \lesseqgtr 0 \quad \text{for} \quad \xi\Lambda \gtrless -\frac{1}{4}\mu^2. \tag{26}$$

Therefore the solution $\Phi = 0$ is unstable or stable for $\xi\Lambda$ is greater or smaller than $-\mu^2/4$. The condition $\xi\Lambda > -\mu^2/4$ is the same as the one for $v^2 > 0$, provided $\lambda > -\kappa\xi\mu^2$, as shown in Eq. (20). On the other hand, for $\Phi_{(c)} = \pm v$,

$$(\delta\Phi)^{;\gamma}{}_{;\gamma} + 2\left(1 + 6\xi^2 v^2\right)^{-1}\left[\frac{\kappa\xi}{1+\kappa\xi v^2}\left(\mu^2 - \lambda v^2\right) + \lambda\right] v^2 \delta\Phi = 0, \tag{27}$$

where

$$2\left(1 + 6\xi^2 v^2\right)^{-1}\left[\frac{\kappa\xi}{1+\kappa\xi v^2}\left(\mu^2 - \lambda v^2\right) + \lambda\right] v^2 > 0 \quad \text{for} \quad \lambda > -\kappa\xi\mu^2. \tag{28}$$

Therefore the solutions $\Phi = \pm v$ are stable for $\lambda > -\kappa\xi\mu^2$. (This condition has appeared in Eq. (20), and another condition $\xi\Lambda > -\mu^2/4$ should also be fulfilled in order to ensure the existence of the solutions $\Phi = \pm v$.)

Consequently, we may conclude that, provided $\lambda > -\kappa\xi\mu^2$, for $\xi\Lambda < \mu^2/4$, there is only one constant solution, $\Phi = 0$, which is stable and corresponding to one true vacuum, while for $\xi\Lambda > \mu^2/4$, there are one unstable constant solution, $\Phi = 0$, and two stable constant solutions, $\Phi = \pm v$, corresponding to two true vacua.

4 Discussion

We have shown that the potential $V_{\mathcal{R}}(\Phi)$, including the non-minimal coupling with gravity and the potential terms used in the standard scenario of the spontaneous symmetry breaking, does entail multiple true vacua, and hence be able to induce spontaneous symmetry breaking, under suitable conditions. In particular, as shown in Eqs. (20), (26), and (28), the conditions

$$\left\{\begin{array}{lcl} \kappa & > & 0 \\ \xi & > & 0 \\ \xi\Lambda & > & -\mu^2/4 \\ \lambda & > & -\kappa\xi\mu^2 \end{array}\right. \tag{29}$$

should be fulfilled. It is interesting to note that the spontaneous symmetry breaking can still be achieved even for a negative coupling constant λ, as long as it is greater than $-\kappa\xi\mu^2$. This is peculiar and quite different from the standard scenario of the spontaneous symmetry breaking.

Accompanying the spontaneous symmetry breaking, under the influence of the non-minimal coupling with gravity, both the gravitational constant G (also κ) and cosmological constant Λ will undergo a "phase transition":

$$8\pi G \equiv \kappa \xrightarrow{\text{SSB}} 8\pi G_{eff} \equiv \kappa_{eff} = \left(\frac{\mu^2 + \lambda/\kappa\xi}{4\xi\Lambda + 2\mu^2 + \lambda/\kappa\xi}\right)\kappa \tag{30}$$

$$\Lambda \xrightarrow{\text{SSB}} \Lambda_{eff} = \frac{\lambda\Lambda - \kappa\mu^4/4}{\lambda + \kappa\xi\mu^2} \tag{31}$$

when the scalar field Φ rolls down to one of the true vacua from the origin. It is interesting to see the large $\xi\Lambda$ limit of κ_{eff}:

$$\xi\Lambda \gg \mu^2, \frac{\lambda}{\kappa\xi}\ , \qquad \kappa_{eff} \simeq \left(\frac{\mu^2 + \lambda/(\kappa\xi)}{4\xi\Lambda}\right)\kappa \ll \kappa.$$

Consequently, the effective gravitational constant G_{eff} (also κ_{eff}) after the spontaneous symmetry breaking can be as small as desired for suitable values of the parameters: μ, λ, ξ, and Λ, even though the original gravitational constant κ could be arbitrarily large. (A similar phenomenon has been pointed out in the "induced gravity" model [15].)

As for the effective cosmological constant Λ_{eff} in Eq. (31), we can see that it will be negative and independent of the original cosmological constant Λ when the coupling constant λ vanishes:

$$\Lambda_{eff} \longrightarrow -\frac{\mu^2}{4\xi} \quad \text{as} \quad \lambda \longrightarrow 0, \tag{32}$$

which has been pointed out in our previous work [14]. Consequently, for $\lambda = 0$, the value of effective cosmological constant Λ_{eff}, after the phase transition, is controlled only by the parameters: μ^2 and ξ. This particular feature might give some hint for dealing with the cosmological constant problem. In addition, if we require a comparatively small effective cosmological constant indicated by various astronomical observations, for instance, the supernova distance measurements, we need to fine tune the parameters: μ and λ, and the original cosmological constant Λ, such that the condition

$$\frac{1}{\kappa}\Lambda \simeq \frac{\mu^4}{4\lambda} \tag{33}$$

is fulfilled. We note that the above condition, where the non-minimal coupling with gravity is involved, is exactly the same as the one in Eq. (2), which is for the case in a flat spacetime. It is somewhat unexpected for us that the fine-tuning condition is unchanged both qualitatively and quantitatively even after introducing the non-minimal coupling.

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